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Simplification of higher-twist evolution in the large N_c limit: why and why not

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Abstract. Working in the light-cone gauge, we find a simple procedure to calculate the autonomous one-loop Q^2 evolution of the twist-three part of the nucleon $g_T(x,Q^2)$ structure function in the large-N_c limit. Our approach allows us to investigate the possibility of a similar large-N_c simplification for other higher-twist evolutions. In particular, we show that it does not occur for the twist-four part of the $f_4(x,Q^2)$, $g_3(x,Q^2)$ and $h_3(x,Q^2)$ distributions. We also argue that the simplification of the twist-three evolution does not persist beyond one loop.

Feynman's parton model of incoherent parton scattering provides a transparent picture of what happens in a broad class of high-energy scattering processes. Modulo field theoretical logarithms, the parton model can be derived in quantum chromodynamics (QCD) in the form of factorization theorems [1]. Better yet, QCD allows us to go beyond the naive parton model by consistently including the effects of the parton transverse momentum and coherent parton scattering. A simple example of coherent parton scattering is the interference of a single quark with a quark *and* a gluon in a nucleon target. To describe this phenomenon, it is necessary to introduce a three-parton light-cone correlation function

$$M^\alpha(x,y,Q^2) = \int \frac{d\lambda}{2\pi}\frac{d\mu}{2\pi} e^{i\lambda x} e^{i\mu(y-x)} \langle PS|\bar\psi(0) iD^\alpha(\mu n)\psi(\lambda n)|PS\rangle \ , \tag{1}$$

where n is a light-cone vector, ψ a quark field, and $|PS\rangle$ the nucleon state. The general parton correlations involve more than one Feynman variable, and hence their scale (Q^2) evolution is more complicated than the usual Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations for the Feynman parton densities. Technically, the complication arises from the so-called higher-twist part of the correlations.

Experimental study of parton correlations is challenging for a number of reasons. One is the lack of processes in which all Feynman variables in a parton correlation can be kinematically controlled. For instance, in polarized lepton-nucleon deep-inelastic scattering (DIS), one can measure the structure function $g_T(x,Q^2)$. In the Bjorken limit, $g_T(x,Q^2)$ is related to a y-moment of the above correlation function. Since a moment of $M_\alpha(x,y,Q^2)$ does not evolve autonomously, knowing the entire $g_T(x,Q^2)$ at one scale is not sufficient to calculate it at another. This makes an analysis of $g_T(x,Q^2)$ data at different scales difficult.

Several years ago, Ali, Braun, and Hiller (ABH) [2] made a remarkable discovery that in the limit of the large number of color N_c, the twist-three part of $g_T(x,Q^2)$ does evolve autonomously at the one-loop level. The result has since been widely used in model calculations and analyses of experimental data [3]. More recently similar results have been found for the evolutions of other twist-three functions $h_L(x,Q^2)$ and $e(x,Q^2)$[4]. Given the practical importance of the ABH result, a deeper understanding of the large N_c simplification is clearly desirable. Moreover, it is interesting to investigate the possibility of a similar simplification at two or more loops and for analogous twist-four correlations.

In this paper we calculate directly the large-N_c evolution of $g_T(x,Q^2)$ in the light-cone gauge. We find that the autonomy of the twist-three evolution arises from a special property of one particular Feynman diagram. Since this property is independent of the γ-matrix structure of the composite operators inserted, the ABH result generalizes immediately to the twist-three parts of $h_L(x,Q^2)$ and $e(x,Q^2)$. Unfortunately, for various reasons we shall explain, there is no similar large-N_c simplification for twist-four functions, nor for $g_2(x,Q^2)$ beyond one loop.

We begin our discussion with a brief introduction to the $g_T(x,Q^2)$ structure function of the nucleon. In inclusive DIS, all information about the nucleon structure is summarized in the following hadron tensor,

$$W^{\mu\nu}(P,S,q) = \frac{1}{4\pi}\int d^4\xi\, e^{iq\cdot\xi}\langle PS|[J_\mu(\xi),J_\nu(0)]|PS\rangle \ , \tag{2}$$

where $J^\mu = \sum_q e_q^2\bar\psi_q\gamma^\mu\psi_q$ is the electromagnetic current and q is the spacelike virtual photon momentum. The antisymmetric part of the hadron tensor, $W^{[\mu\nu]}$, is polarization-dependent and can be characterized in terms of the two structure functions $g_1(x_B, Q^2)$ and $g_2(x_B, Q^2)$:

$$W^{[\mu\nu]} = -i\epsilon^{\mu\nu\alpha\beta} q_\alpha \left(S_\beta \frac{g_1(x_B, Q^2)}{\nu} + [\nu S_\beta - (S \cdot q) P_\beta] \frac{g_2(x_B, Q^2)}{\nu^2} \right), \quad (3)$$

where we have chosen the kinematic factors so that $g_1(x_B, Q^2)$ and $g_2(x_B, Q^2)$ survive the scaling limit $Q^2 = -q^2 \to \infty$, $\nu = P \cdot q \to \infty$ and $x_B = Q^2/2\nu =$ finite. In Feynman's parton model, $g_1(x_B, Q^2)$ is related to the parton helicity density $\Delta q_a(x, Q^2)$

$$g_1(x_B, Q^2) = \frac{1}{2} \sum_a e_a^2 \left[\Delta q_a(x_B, Q^2) + \Delta q_a(-x_B, Q^2) \right], \quad (4)$$

where e_a is the electric charge and a sums over light quark species.

The structure function $g_2(x_B, Q^2)$, however, does not have a simple parton model interpretation. Defining $g_T(x_B, Q^2) = g_1(x_B, Q^2) + g_2(x_B, Q^2)$, an operator-product-expansion analysis yields [5]

$$g_T(x_B, Q^2) = \frac{1}{2} \sum_a e_a^2 \left(\Delta q_{Ta}(x_B, Q^2) + \Delta q_{Ta}(-x_B, Q^2) \right), \quad (5)$$

where we have neglected all power and radiative corrections and

$$\Delta q_{Ta}(x, Q^2) = \frac{1}{2M} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS_\perp | \bar{\psi}_a(0) \gamma^\perp \gamma_5 \psi_a(\lambda n) | PS_\perp \rangle . \quad (6)$$

The trouble with a parton model interpretation of $\Delta q_{Ta}(x, Q^2)$ can easily be seen in light-front quantization in which only the "good" component of the Dirac field $\psi_+ = P_+\psi$ has a simple Fock expansion ($P_\pm = \gamma^\mp\gamma^\pm/2$, $\gamma^\pm = (\gamma^0 \pm \gamma^3)/\sqrt{2}$), whereas the "bad" component $\psi_- = P_-\psi$ is constrained by the following equation of motion

$$\psi_-(\lambda n) = -\frac{1}{2} \frac{1}{in \cdot \partial} \not{n} i \not{D}_\perp(\lambda n) \psi_+(\lambda n) . \quad (7)$$

[In some sense ψ_- represents a quark-gluon composite.] Unlike $\Delta q_a(x, Q^2)$, $\Delta q_{Ta}(x, Q^2)$ contains a bad component because of the $\gamma^\perp$.

For the same reason, the scale evolution of $\Delta q_{Ta}(x, Q^2)$ is now more intricate than that of $\Delta q_a(x, Q^2)$. Its n-th moment is written

$$\int_{-1}^{1} \Delta q_{Ta}(x, Q^2) x^n dx = \frac{1}{2M} n_{\mu_1} \cdots n_{\mu_n} \langle PS_\perp | \theta^{\perp(\mu_1 \cdots \mu_n)} | PS_\perp \rangle, \quad (8)$$

where $\theta^{\sigma(\mu_1 \cdots \mu_n)} = \bar{\psi}\gamma^\sigma iD^{(\mu_1} \cdots iD^{\mu_n)}\psi$, with $(\mu_1 \cdots \mu_n)$ indicating symmetrization of the indices and removal of the traces. The θ-operator contains both twist-two $\theta^{(\sigma\mu_1 \cdots \mu_n)}$ (totally symmetric and traceless) and twist-three $\theta^{[\sigma(\mu_1]\mu_2 \cdots \mu_n)}$ (mixed symmetric and traceless) contributions, where $[\sigma\mu_1]$ denotes antisymmetrization. It turns out, however, that for a given symmetry structure there are multiple twist-three operators. In fact, a complete basis of these operators was first identified in [6],

$$R_i^n = \bar{\psi} iD^{(\mu_1} \cdots iD^{\mu_{i-1}} (-ig) F^{\sigma\mu_i} iD^{\mu_{i+1}} \cdots iD^{\mu_{n-1}} \gamma^{\mu_n)} \gamma_5 \psi$$
$$S_i^n = \bar{\psi} iD^{(\mu_1} \cdots iD^{\mu_{i-1}} g\tilde{F}^{\sigma\mu_i} iD^{\mu_{i+1}} \cdots iD^{\mu_{n-1}} \gamma^{\mu_n)} \psi , \quad (9)$$

where $i = 1, ..., n-1$. The operator $\theta^{[\sigma(\mu_1]\mu_2 \cdots \mu_n)}$ is just a special linear combination of them,

$$\theta^{[\sigma(\mu_1]\mu_2 \cdots \mu_n)} = \frac{1}{2(n+1)} \sum_{i=1}^{n-1} (n-i)(R_i^n - R_{n-i}^n + S_i^n + S_{n-i}^n). \quad (10)$$

The anomalous dimension matrix in the above operator basis was first worked out by Bukhvostov et al. and later reproduced by a number of authors with different methods[7]. The result is what one would generally expect. To evolve the matrix element of $\theta^{[\sigma(\mu_1]\mu_2 \cdots \mu_n)}$, it is not enough just to know it at an initial scale—one must know all the matrix elements of $W_i^n = R_i^n - R_{n-i}^n + S_i^n + S_{n-i}^n$ there.

By studying the anomalous dimension matrix in the large N_c limit, Ali, Braun and Hiller found that the eigenvector corresponding to the lowest eigenvalue is just the linear combination of twist-three operators on the right-hand side of (10). In other words, the twist-three part of $\Delta q_{Ta}(x, Q^2)$ evolves autonomously in this limit. To better understand ABH's result, we calculate the large-N_c evolution of $\Delta q_{Ta}(x, Q^2)$ directly. We start with the mixed-twist operator $\theta^{\sigma(\mu_1\mu_2 \cdots \mu_n)}$ in (8) and look for possible divergences when inserted in multi-point Green's functions. To reduce the number of Feynman diagrams, we choose the light-cone gauge $A^+ = 0$ and take the $\perp + \cdots +$ component of the θ-operator. Let's call the resulting operator $\theta_n \equiv \bar{\psi}\gamma^\perp\gamma_5(i\partial^+)^n\psi$, and its twist-two and twist-three parts θ_{n2} and θ_{n3}, respectively. The Feyman rule for θ_n is simply $\gamma^\perp\gamma^5(k^+)^n$, where k is the momentum of the quark.

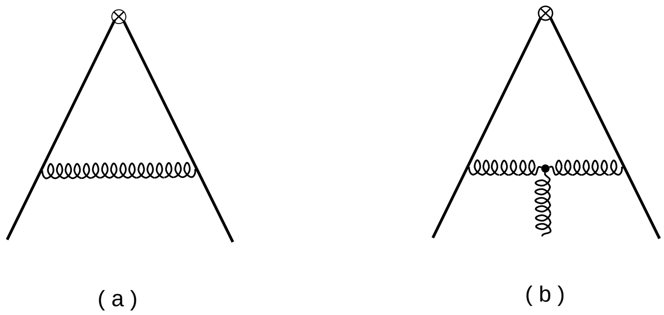
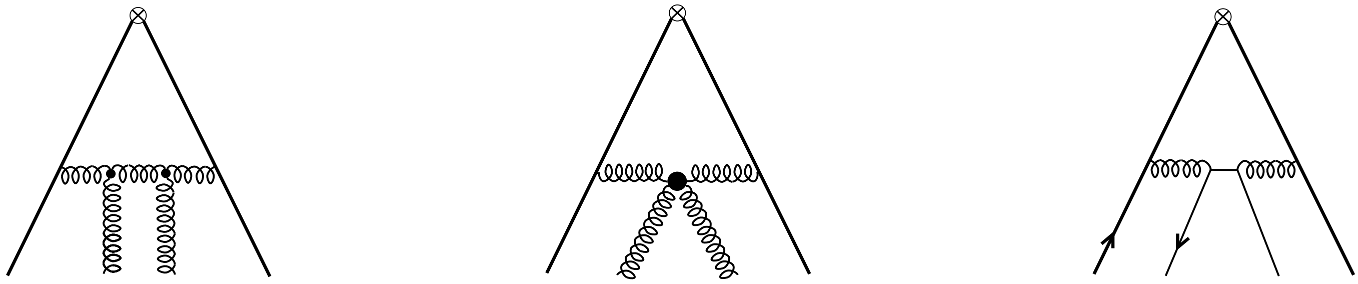


Fig. 1a,b. Two and three-point 1PI Feynman diagrams contributing to the evolution of θ_n in the large N_c limit

By light-cone power counting, we need only consider two- and three-point functions. Since the external lines carry color, we must ask what type of diagrams dominates the large N_c limit. The simple rule we find is that when all external lines are drawn to one point (infinity), the



(a) (b) (c)

Fig. 2a–c. Four-point 1PI Feynman diagrams contributing to the evolution of $\hat{O}$ in the large N_c limit

planer diagrams are leading. All one-particle-irreducible (1PI) leading diagrams with one θ insertion are shown in Fig. 1.

The ultraviolet divergences in the two point Green's function can obviously be subtracted with the matrix element of θ_n itself. The only diagram in which the divergences may not be subtracted by θ_n is Fig. 1b. An explicit calculation shows that the ultraviolet divergences correspond to the following local operator:

$$\begin{aligned} &\frac{1}{2}C_A\frac{g^2}{8\pi^2}\ln Q^2 \qquad (11)\\ &\cdot\Bigg[-\frac{1}{(n+2)}\sum_{i=0}^{n-1}\bar{\psi}\ \not{n}\gamma_5(i\partial^+)^i iD^\perp(i\partial^+)^{n-1-i}\psi \\ &+\left(\sum_{i=1}^{n+1}\frac{1}{i}-\frac{1}{2(n+1)}\right)\Big(\bar{\psi}i\not{D}_\perp\ \not{n}\gamma^\perp\gamma_5(i\partial^+)^{n-1}\psi \\ &\qquad +\bar{\psi}(i\partial^+)^{n-1}\gamma^\perp\gamma_5\ \not{n}i\not{D}_\perp\psi\Big)\Bigg]\,, \end{aligned}$$

where we have neglected the contributions of light-cone singularities which will be cancelled eventually. Notice that the first term is present in the twist-two operator

$$\theta_{n2}=\frac{1}{n+1}\left(\bar{\psi}\gamma^\perp\gamma_5(i\partial^+)^n\psi+\sum_{i=0}^{n-1}\bar{\psi}\gamma^+\gamma_5(i\partial^+)^i iD^\perp(i\partial^+)^{n-i-1}\psi\right), \qquad (12)$$

and the remaining two terms can be converted to θ_n by using the equation of motion in (7). Thus we easily arrive at the ABH conclusion that θ_{n3} evolves autonomously in the large-N_c limit.

Including the contribution from Fig. 1a as well as the one-particle-reducible ones that cannot be neglected in the light-cone gauge, we obtain the following equation,

$$\begin{aligned} &\frac{d\theta_n}{d\ln Q^2}= \qquad (13)\\ &\frac{\alpha_s(Q^2)}{2\pi}\frac{N_c}{2}\left[\frac{n+1}{n+2}\theta_{n2}+\left(-2\sum_{i=1}^{n+1}\frac{1}{i}+\frac{1}{n+1}+\frac{1}{2}\right)\theta_n\right]. \end{aligned}$$

Separating out the twist-two and twist-three parts, we not only recover the well-known twist-two evolution, but also the twist-three result

$$\frac{d\theta_{n3}}{\ln Q^2}=\frac{\alpha_s(Q^2)}{2\pi}\left(-2\sum_{i=1}^{n+1}\frac{1}{i}+\frac{1}{n+1}+\frac{1}{2}\right)\theta_{n3}, \qquad (14)$$

which is identical to the result in [2].

It is quite clear that the i-independence of the coefficients in the sum of (12) is the key for the autonomous evolution of θ_{n3}. On the other hand, this property is not totally unexpected if one inspects Fig. 1b more closely. Interpreting this diagram in the coordinate space, we see that the internal gluon propagates *homogeneously* from one quark to the other. By homogeneously, we mean that at any point along the path of the propagation, the gluon behaves exactly the same way, except, of course, at the points where the gluon and quarks interact. Now the spatial location of the interaction with the external gluon determines the number of derivatives before and after the gluon field in the subtraction operator. Since the internal gluon propagation is homogenous, different locations of the triple-gluon vertex should produce similar physical effects. Therefore, the coefficients of the subtraction operators $\bar{\psi}\ \not{n}\gamma_5(i\partial^+)^i iD^\perp(i\partial^+)^{n-1-i}\psi$ should be independent of i. On the other hand, the two extra terms in (12) correspond to the triple-gluon vertex just next to the external quark lines, where the homogeneity is lost.

Since the homogeneous property of the internal gluon line is independent of the gamma matrix structure of the operator inserted, we conclude that the other twist-three distributions $e(x,Q^2)$ and $h_L(x,Q^2)$ evolve also autonomously in the large N_c limit. A quick calculation confirms the evolution equations found in [4].

Encouraged by the success of the above approach, we apply it to the analogous twist-four evolution. In [8], the three one-variable distributions $f_4(x,Q^2)$, $g_3(x,Q^2)$ and $h_3(x,Q^2)$ are shown to contain twist-four. For example, $f_4(x)$ is defined as

$$f_4(x)=\frac{1}{M^2}\int\frac{d\lambda}{2\pi}\langle P|\bar{\psi}(0)\gamma^-\psi(\lambda n)|P\rangle\ . \qquad (15)$$

It was shown in [9] that $f_4(x, Q^2)$ contributes to the $1/Q^2$ term of the longitudinal scaling function F_L of the nucleon

$$F_L(x_B, Q^2) = \frac{2x_B^2 M^2}{Q^2} \sum_a e_a^2 f_{4a}(x_B, Q^2)\ , \tag{16}$$

where we have neglected higher-order radiative corrections. Here, autonomous evolution of $f_4(x, Q^2)$ would simplify the analysis of F_L data immensely.

In the large N_c limit, we consider one insertion of the operator $\hat{O} = \bar\psi\gamma^-(i\partial^+)^n\psi$ into two-, three- and four-point Green's functions. At one-loop order, the 1PI two- and three-point diagrams are identical to those in Fig. 1 and the 1PI four-point diagrams are shown in Fig. 2. Only the three and four point diagrams can potentially destroy the autonomous evolution of $\hat{O}$. Let us start with Fig. 2a. One of the divergent contributions from this diagram introduces the following local subtraction

$$\sum_i \bar\psi i \not{D}_\perp \not{n}(i\partial^+)^i i \not{D}_\perp (i\partial^+)^{n-i-2}\psi + \text{h.c.} \tag{17}$$

where all the coefficients are independent of i again because of the homogeneity of the gluon propagator. Using the equation of motion, we can write this as

$$\sum_i \bar\psi (i\partial^+)^i i \not{D}_\perp (i\partial^+)^{n-i-2}\psi + \text{h.c.} \tag{18}$$

Since this operator cannot be reduced to either the twist-two or twist-four part of $\hat{O}$, the evolution of the latter cannot be autonomous unless this contribution is cancelled by other diagrams. The only other diagram containing the same divergence structure is Fig. 1b with an insertion of $\hat{O}$. Unfortunately, our explicit calculation did not produce this cancellation. The same phenomenon occurs for the twist-four part of $g_3(x, Q^2)$ and $h_3(x, Q^2)$.

Thus, the large N_c simplification seems to happen only for the evolution of the twist-three part of $g_T(x, Q^2)$, $h_L(x, Q^2)$ and $e(x, Q^2)$. Does it happen for them at two and higher loops? In Fig. 3, we show two examples of Feynman diagrams that we suspect break the autonomy of the θ_{3n}-evolution, i.e., they may contain divergences that cannot be subtracted by θ_{n2} and θ_{n3} only. Our suspicion is based on the inhomogeneity of the gluon progator. The internal gluon that propagates from one quark to another has different wavelengths in the different parts of the propagation. Its interaction with the external gluon is different at different spatial locations. Thus the subtraction operators have different coefficients depending on the number of derivatives before and after the external gluon field. An explicit calculation of Fig. 3a confirms our suspicion.

This leaves us with only one possibility for autonomous two-loop evolution of θ_{n3}: the unwanted structures cancel in the sum of all large-N_c two-loop diagrams. Calculating all those diagrams is a big task. However, even without an explicit calculation, we do not expect the cancellation to happen. The fundamental reason is that large N_c represents only a selection of a subset of Feynman diagrams, whereas the result of an individual diagram is independent of the large-N_c limit. Cancellations of a structure do not happen among Feynman diagrams unless there is a symmetry.

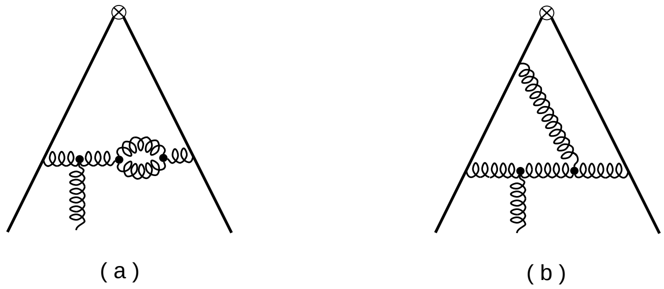


Fig. 3a,b. Some two-loop 1PI Feynman diagrams that might break the autonomy of the θ_n in the large N_c limit

Therefore we conclude that the autonomy of one-loop evolution for a set of special twist-three distributions at large N_c seems accidental. In the light-cone gauge, it can be easily traced to a special property of Fig. 1b. The simplification does not happen for the analogous twist-four distributions at one loop, nor for those twist-three distributions at two or higher loops. Nonetheless, the discovery of Ali, Braun, and Hiller remains as a significant step forward in the study of the $g_2(x, Q^2)$ structure function. Without the autonomous one-loop evolution, an analysis of experimental data on the twist-three contribution would be severely constrained.

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